



Cosmic Origins: Measuring Parallax Using Similar Triangles

Lesson Plan for Grades: 10th grade Length of Lesson: 90 mins
Authored by: UT Environmental Science Institute Date created: 05/10/2017
Subject area/course: <ul style="list-style-type: none">● Geometry / Astronomy
Materials: <ul style="list-style-type: none">● Set of HD colored pictures for Engage or student accessible file with all pictures (1 set per group of 2-3 students)● Any object in classroom (1 per class)● Beginner's telescope (should be <\$20) (suggested amount is 1 per group of 6-9 students, with assumption that groups will need to share this material)● Long (over 300ft or 100m) measured tape roll (1 per group of 3 students)● Easel or stand with flat background of paper sized space (1 per group of 3 students)● Graph paper to cover easel/background (1 per group of 3 students)● 1 roll of clear tape (to use in sticking graph paper to easel) (1 per class)● Small objects like pennies, washers, pencils, erasers, etc. (2 different ones per group of 3 students)● Support from which to hang the small object (like a tripod, music stand, secondary easel, etc.)● Thin string to hang object (1 per group of 3 students)● Explore Parallax Handout (1 per student)● Relatively empty football field (or empty space of about 100m distance to conduct activity, and enough width for all groups to be able to set up their own experiment)● Teacher access to computer with Internet (for playing Elaborate clip) (Teacher's own)● Teacher access to projector (to show Elaborate clip) (Teacher's own)
TEKS/SEs: §111.41. Geometry, Adopted 2012 (One Credit). (c) Knowledge and skills. (1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to: <ul style="list-style-type: none">(A) apply mathematics to problems arising in everyday life, society, and the workplace;(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate; (5) Logical argument and constructions. The student uses constructions to validate conjectures about geometric figures. The student is expected to: <ul style="list-style-type: none">(C) use the constructions of congruent segments, congruent angles, angle bisectors, and perpendicular bisectors to make conjectures about geometric relationships;



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- (7) Similarity, proof, and trigonometry. The student uses the process skills in applying similarity to solve problems. The student is expected to:
- (A) apply the definition of similarity in terms of a dilation to identify similar figures and their proportional sides and the congruent corresponding angles; and
 - (B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.

§112.33. Astronomy, Beginning with School Year 2010-2011 (One Credit).

- (c) Knowledge and skills.
- (2) Scientific processes. The student uses scientific methods during laboratory and field investigations. The student is expected to:
- (F) collect data and make measurements with accuracy and precision;
 - (G) organize, analyze, evaluate, make inferences, and predict trends from data, including making new revised hypotheses when appropriate;
 - (H) communicate valid conclusions in writing, oral presentations, and through collaborative projects; and
 - (I) use astronomical technology such as telescopes, binoculars, sextants, computers, and software.
- (3) Scientific processes. The student uses critical thinking, scientific reasoning, and problem solving to make informed decisions within and outside the classroom. The student is expected to:
- (E) describe the connection between astronomy and future careers.

§112.36. Earth and Space Science, Beginning with School Year 2010-2011 (One Credit).

- (c) Knowledge and skills.
- (2) Scientific processes. The student uses scientific methods during laboratory and field investigations. The student is expected to:
- (G) organize, analyze, evaluate, make inferences, and predict trends from data;
 - (H) use mathematical procedures such as algebra, statistics, scientific notation, and significant figures to analyze data using the International System (SI) units; and
 - (I) communicate valid conclusions supported by data using several formats such as technical reports, lab reports, labeled drawings, graphic organizers, journals, presentations, and technical posters.

Lesson objective(s): Students Will Be Able To (SWBAT):

- sort different cosmic images based on similar properties such as color, mass, and distance
- calculate different distances between objects across large distances, using concepts relating to similar triangles and proportions
- evaluate differences between calculations and actual measurements for accuracy

Differentiation strategies to meet diverse learner needs:

- The teacher should ask students whether they prefer to read or watch videos to learn about concepts; then have students learn in their preferred learning style. However, the teacher may assign students certain methods to improve their skills. For example, if a student prefers reading, teachers may have them watch a video and take notes to improve their listening skills.



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- ELL students and students with learning disabilities should have multiple forms of instruction including visual and written instruction sheets as well as a verbal instruction and demonstration.

ENGAGEMENT (5 minutes)

- The teacher will give pictures to groups of 2-3 students and instruct the class to find a way to sort the images and actually sort all their images using their own method. [2-3 mins]
 - These images are given in later pages of Lesson Plan and should be printed double sided so the correct description for each image is on the back.
- The teacher will then have students share out loud the various ways they arranged and sorted the pictures. These different methods can be written down on the board as students discuss possible sorting methods. [2-3 minutes]
- *Possible student questions:*
 - Should we sort only based on one variable, or can we sort the pictures further?
 - You can sort however you feel is best, just remember that you will need to explain your reasoning afterwards.
 - Is there only one possible way to sort?
 - No, there are many ways.
 - Are there other ways to sort these pictures (besides the given variables)?
 - Yes, can you hypothesize on some for me?
 - Expected responses: temperature, size, age, brightness, etc.
- Two examples of how students may sort the pictures are listed below (**teacher answer key**):
 - **Sorted by distance: (1 light year is 9.461×10^{12} kilometers)**
 1. Stellar Mass Black Hole (~30 kilometers across)
 2. Moon (~3400 kilometers)
 3. Jupiter (~140,000 kilometers)
 4. Sun (~1.4 million kilometers)
 5. HL Tau Star Forming Disk (~0.002 light years across, 18.9 billion kilometers)
 6. Crab Nebula (~5.5 light years across)
 7. Pleiades Open Star Cluster (~ 16 light years across)
 8. M5 Globular Star Cluster (~160 light years across)
 9. Pinwheel Spiral Galaxy (~170 light years across)
 10. Sombrero Galaxy (~ 50,000 light years across)
 11. Abell 370 Galaxy Cluster (~6 million light years across)
 12. Large Scale Structure of the Universe (~3 billion light years across)
 - **Sorted by Size: (compared to the mass of the Sun)**
 1. Moon (~ 40 billionths)
 2. Jupiter (1/1000 mass of Sun)
 3. Sun (=)
 4. HL Tau Star Forming Disk (1.1 times)
 5. Stellar Mass Black Hole (5-10 times)
 6. Crab Nebula (10 times)
 7. Pleiades Open Star Cluster (~ 800 times)
 8. M5 Globular Star Cluster (1 million times)
 9. Pinwheel Spiral Galaxy (100 billion times)
 10. Sombrero Galaxy (~ 800 billion times)
 11. Abell 370 Galaxy Cluster (~1014 times)
 12. Large Scale Structure



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EXPLORATION (40 minutes)

In-class pre-activity: [should take about 7-10 minutes]

- The teacher will first conduct an intro to the experimental concept by having students place a pencil or finger in front of any object in the classroom (students get to pick their own object).
 - The object must be farther than 3 feet away from where the student is, and small enough that the student can cover the object with their finger (just using their eyes, from that distance).
- The students will find an appropriate object, close one eye, and place their finger in between themselves and the object (so that the object appears to be covered by their finger).
- The teacher will then instruct the students to switch eyes (close the first eye, and open their other one), and then quietly (in their heads) observe what happens. The teacher should emphasize to students to keep their finger steady between the two observations.
- The teacher will then ask students what happened/changed between the two observations.
- The teacher will guide the class in a quick discussion hypothesizing on what made the covered object appear again (just by switching eyes).
 - The teacher should elicit students to bring up the change in distance from eye 1 to the object, to eye 2 to the object as a possibility for why the object appeared to move (due to the concept of parallax).
 - The teacher can have students come up to the front of the classroom and draw this out on the board (using triangles) as a visual aid to the discussion.

Outdoors Main Activity: [30-33 mins]

- Next, the teacher will assign students to groups of 3-4 and hand out supplies (telescopes, string, easel, measuring tape, graph paper, tape, Explore Handout, etc.). The teacher will then take the students outside to a large enough field (should be spacious and relatively empty). The teacher will set up one graph “paper-ease-object-string-tripod-telescope” apparatus to show students (as a visual demo) of how the experiment should be set up. [2-5 mins, can be done prior to the lesson to save time]
- A diagram of the experimental set is shown in Figure 1. There are 2 different objects hanging by a string attached to the tripod/music stand (at different distances from the easel), and the grid or graph paper is taped to the flat side of the easel (make sure to keep it exactly perpendicular to the ground).
- In-depth procedures for this activity are listed on the Exploration Activity page.
- Teachers should allow for student-directed learning during this time, but should also be highly engaged while students are performing activities, asking students questions one-on-one or while they are working in groups. This method of probing should act as a formative assessment. Example probing questions that the teacher can ask include:
 - What are you measuring?
 - Why is b' significant?
 - Between the two objects, what differences are there in measured values?
 - What contributes to these differences?
 - What are some possible sources of error?
 - Do you think every group will get the same answer? Why or why not?

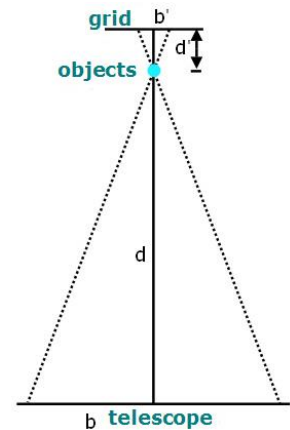


Figure 1. Outdoors activity experimental set up (Image source: sciencebuddies.org)



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- How does this activity relate back to the pre-activity we did using our eyes in the classroom?

EXPLANATION (15 minutes)

- Students will explain the distances they found from both calculations and actual measurements, as well as % error calculated, and parallax calculations.
- Students will explain what may have led to differences between the measured and calculated distances.
- After each group presents their experimental findings, the teacher will lead a small mini-lecture to explain parallax as it relates to the Explore activity. The teacher will then ask students to brainstorm other possible “far away objects” the students can calculate distances and parallaxes for (the teacher will probe for responses looking at large-scale objects like moons, other planets, nearby stars, etc.).
- The teacher will write each measurement and calculation from each group on the board. This will tie into the Elaborate activity.

ELABORATION (30 minutes)

- The teacher will have students watch a short animated clip of a nearby star’s parallax to the Earth as it rotates around the Sun in an orbit: www.astronomy.ohio-state.edu/~pogge/Ast162/Movies/parallax.gif and will actively observe students while the clip plays on loop for 3-5 times. Then, after the students have looked at the clip on their own, the teacher will explain what each thing on the clip signifies: for example, the top view is a view of the night sky from Earth, with the red dot the sky (that looks like it moves back and forth due to parallax) representing the nearby star shown on the bottom view. The dates show changes through time, while the bottom view shows Earth’s path orbiting the Sun, while the nearby star is at 1 unit and 2 units (of distance, shown by the scale bar) and the red line shows one side of the distance to the star. [5 mins]
- The teacher will then play the clip several more times and have students debate the question, “In astronomy, can scientists use parallax on super far away stars? Why or why not?”
- The students will be encouraged to use whatever methods they need to help explain their points during the class debate. The teacher can act as a guide as well as play the opposing side to probe for student understanding.
- Through verbal teacher scaffolding during the debate (see www.astronomy.ohio-state.edu/~pogge/Ast162/Unit1/distances.html for more background information for the teacher’s preparation), the teacher will then guide students to come to the conclusion that “No, parallax cannot be used for faraway stars from Earth, because the angle will be too small to calculate accurately.” [This entire activity will take approximately 15-20 mins]
- The teacher should show a simple formula relating parallax to distance ($D = 1/P$) and ask students to discuss with a Turn and Talk (TNT) with their table partners/desk neighbors what methods astronomers can use instead to calculate parallaxes of stars that are super far away from Earth. The entire class will then discuss as a whole after the TNT technique. [5-10 mins]
 - Expected student responses: using other objects in space as the “Earth” for observing parallax to the object, in order to cut down on the distance to the star (i.e. satellites, or the Moon or Mars)
 - Using other techniques or computer software programs (to model stars’ distances based on relative characteristics such as color, brightness, etc.) unrelated to geometry (or similar triangles in particular).
- In case of extra left-over time, have students debate on whether the orbit of Earth as well as Earth’s rotation and tilt can affect (or not) the parallax measured to a nearby star. Have



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students also discuss other possible variables that may affect the calculation (such as the curvature of Earth, the time of year, etc.). [5-10 mins]

- This debate should be open-ended and as a follow-up homework assignment, the teacher can have students pick one variable (such as Earth's orbit) and research whether it affects the distance to a star (determined by parallax). They can write a 1-2 paragraph summary detailing the resource they used and what they learned about their variables importance/unimportance.
- Vocabulary and concepts that will be introduced/discussed in this part of the lesson:
 - Parallax: the angular displacement of an observed object due to changes in the position of the observer
 - Orbit: the curved path of an object around another object
 - Similar triangles: triangles with equal corresponding angles (congruent to each other) and proportionate sides
 - Proportions: having a constant ratio that can be used when making comparisons between different sized objects.
- Similar triangles and proportions can be used not only in astronomy, but in many other fields like geography and geology, where large-scale spatial distances can be inferred from using geometric concepts (using angles and known distances).

EVALUATION (throughout entire lesson)

- Formative Assessment:
 - Students will be actively observed by teacher while they work in groups for the sorting activity (Engage).
 - Students will be actively observed during the activities for the Explore, and the teacher will ask students probing questions to determine student understanding.
 - Students will be actively observed as they explain their experiments (during the Explain) and engage in class discussions about the significance of parallax.
- Summative Assessment:
 - Students will fill out the Explore Handout on Parallax, which the teacher will use to assess student understanding of the concepts discussed.
 - The assigned homework (for the "if remaining time available," final, elaborate activity) can be used as an additional summative assessment that the teacher can use to assess student learning.

SOURCES AND RESOURCES

- Special thanks to Dr. Caitlin Casey for the sorting activity used in the Engagement section.
- **Dr. Caitlin Casey's Hot Science – Cool Talks #117, "Investigating Our Cosmic Origins"**, www.esi.utexas.edu/talk/investigating-our-cosmic-origins/
- **Science Buddies Staff. "Similar Triangles: Using Parallax to Measure Distance."** *Science Buddies*, 28 July 2017, www.sciencebuddies.org/science-fair-projects/project-ideas/Astro_p019/astronomy/similar-triangles-using-parallax-to-measure-distance Accessed 20 Mar. 2019.
- **Trigonometric Parallax Movie (with Explanation)**, www.astronomy.ohio-state.edu/~pogge/Ast162/Movies/parallax.html
- **Science Buddies, image of procedural setup** www.sciencebuddies.org/science-fair-projects/project-ideas/Astro_p019/astronomy/similar-triangles-using-parallax-to-measure-distance#procedure



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ENGAGEMENT ACTIVITY

Purpose: To engage students' attention on how stars and other cosmic features (galaxies, black holes, universes) can be grouped together based on certain similar characteristics (distance, mass, color, etc.)

Materials: Set of pictures (shown below)

Safety Information: Students must work together in groups while remaining respectful of their peers and their ideas.

Procedure: Described in previous pages. Print handouts in the following pages, double-sided so the image description is in the back of each picture.



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EXPLORATION ACTIVITIES

Purpose: For students to be able to apply concepts relating to parallax using calculations with similar triangles and astronomical tools like telescopes

Materials: listed above (at the start of the lesson plan)

Safety Information: Students should behave appropriately both in the classroom and outside, following school policies and classroom rules. Students should also take care of classroom equipment (such as easel, telescope, tripod/music stand, etc.). Students should work together in groups while being respectful of peers and their ideas.

Procedure:

(For the **first, in-classroom activity** portion of Explore)

1. The teacher will first conduct an intro to the experimental concept by having students place a pencil or finger in front of any object in the classroom (students get to pick their own object).
 - a. The object must be farther than 3 ft away from where the student is, and small enough that the student can cover the object with their finger (just using their eyes, from that distance).
2. The students will find an appropriate object, close one eye, and place their finger in between themselves and the object (so that the object appears to be covered by their finger).
3. The teacher will then instruct the students to switch eyes (close the first eye, and open their other one), and then quietly (in their heads) observe what happens. The teacher should emphasize to students to keep their finger steady between the two observations.
4. The teacher will then ask students what happened/changed between the two observations.
5. The teacher will guide the class in a quick discussion hypothesizing on what made the covered object appear again (just by switching eyes).
 - a. The teacher should elicit students to bring up the change in distance from eye 1 to the object, to eye 2 to the object as a possibility for why the object appeared to move (due to the concept of parallax).
 - b. The teacher can have students come up to the front of the classroom and draw this out on the board (using triangles) as a visual aid to the discussion.

(For the **second, outside activity** portion of Explore) (from: www.sciencebuddies.org/science-fair-projects/project-ideas/Astro_p019/astronomy/similar-triangles-using-parallax-to-measure-distance)

1. At one end of the field, place the easel with the graph paper attached to it (making sure that it is perpendicular to the ground). Position the graph paper at a convenient height for viewing straight-on with the telescope. Hang two small objects directly in front of the center of the grid at two different distances from the easel (labeled d' on the diagram on the Explore section). Measure and record the distance, d' , for each object.
2. Place your telescope approximately 100 meters away from the easel (keeping it directly in front of the graph paper). The objects should line up with the center of the graph paper



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when centered in the telescope. Mark the center-line of the graph paper, and mark the position of the telescope.

3. Now move the telescope one meter to the left (keeping it at the same height). Center the first object in the telescope, and use the graph paper to measure how far the object is from the center line (this is your b' measurement for the first object). Then center the second object in the telescope, and use the graph paper to measure how far the object is from the center line (this is your b' measurement for the second object).
4. Now move the telescope one meter to the right of the straight-on position. Again, center each object and measure how far it is from the center line.
5. Take the average of your left and right measurements for each object to get your b' value and record it.
6. Use the ratio $s'/b' = s/b$ to calculate the distance s from the telescope to each object. Is the distance between the two objects the same as what you get from your direct measurements?
7. Use measuring tape to measure the distance from the telescope (center position) to the different objects. How does the direct measurement compare to your parallax calculations?



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STUDENT HANDOUT:

Name: _____

Period: _____

Parallax Handout:

PRE-ACTIVITY:

1. What happens when you change eyes (close the other eye you had first opened) to look at your object?

EXPERIMENT PROCEDURES:

2. At one end of the field, **place the easel with the graph paper attached** to it (making sure that it is perpendicular to the ground). Position the graph paper at a convenient height for viewing straight-on with the telescope. **Hang two small objects** directly in front of the center of the grid at two different distances from the easel (labeled d' on the diagram on the Explore section). **Measure and record the distance, d'** , for each object. (INCLUDE YOUR UNITS!)

Object 1: $d' =$ _____

Object 2: $d' =$ _____

3. Place your telescope approximately **100 meters away from the easel** (keeping it directly in front of the graph paper). The objects should **line up with the center of the graph paper** when centered in the telescope. **Mark the center-line** of the graph paper, and **mark the position of the telescope**.
4. Now move the telescope **one meter to the left** (keeping it at the same height). Center the first object in the telescope, and use the graph paper to **measure how far the object is from the center line** (this is your $b1'$ measurement for the first object). Then **center the second object** in the telescope, and **use the graph paper to measure** how far the object is from the center line (this is your $b1'$ measurement for the second object). (INCLUDE YOUR UNITS!)

Object 1: $b1' =$ _____

Object 2: $b1' =$ _____

5. Now **move the telescope one meter to the right** of the straight-on position. Again, **center each object** and **measure how far** it is from the center line.

Object 1: $b2' =$ _____

Object 2: $b2' =$ _____

6. Take the **average** of your left and right measurements **for each object** to get your b' value and record it. Hint: Average (b') = $[b1' + b2']/2$



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Object 1: $b' =$ _____

Object 2: $b' =$ _____

7. Use the ratio $s'/b' = s/b$ to calculate the distance s from the telescope to each object. Is the distance between the two objects the same as what you get from your direct measurements? (SHOW CALCS AND WRITE OUT EXPLANATIONS!)
8. Use measuring tape to measure the distance from the telescope (center position) to the different objects. **How does the direct measurement compare to your parallax calculations?**
9. In your own words, **define parallax and draw out a picture to explain it.**
10. How is **parallax related to** the telescope's **distance** to each of the objects?
11. **Who** would care about **using parallax**? **Why** would this be important for them?
12. Describe any **2 possible sources of error** when you and your group were conducting this experiment, as well as how they may have affected your data/calculations.

Source of Activity and Handout Instructions/Procedures: www.sciencebuddies.org/science-fair-projects/project-ideas/Astro_p019/astronomy/similar-triangles-using-parallax-to-measure-distance



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TEACHER HANDOUT:

Name: ___KEY___

Parallax Handout:

PRE-ACTIVITY:

1. What happens when you change eyes (close the other eye you had first opened) to look at your object?

Object appears to move slightly/a lot (depending on how far away the object is).

EXPERIMENT PROCEDURES:

2. At one end of the field, **place the easel with the graph paper attached** to it (making sure that it is perpendicular to the ground). Position the graph paper at a convenient height for viewing straight-on with the telescope. **Hang two small objects** directly in front of the center of the grid at two different distances from the easel (labeled d' on the diagram on the Explore section). **Measure and record the distance, d'** , for each object. (INCLUDE YOUR UNITS!)

Object 1: $d' =$ ___ *Variable responses* ___

Object 2: $d' =$ ___ *Variable responses* ___

3. Place your telescope approximately **100 meters away from the easel** (keeping it directly in front of the graph paper). The objects should **line up with the center of the graph paper** when centered in the telescope. **Mark the center-line** of the graph paper, and **mark the position of the telescope**.
4. Now move the telescope **one meter to the left** (keeping it at the same height). Center the first object in the telescope, and use the graph paper to **measure how far the object is from the center line** (this is your $b1'$ measurement for the first object). Then **center the second object** in the telescope, and **use the graph paper to measure** how far the object is from the center line (this is your $b1'$ measurement for the second object). (INCLUDE YOUR UNITS!)

Object 1: $b1' =$ ___ *Variable responses* ___

Object 2: $b1' =$ ___ *Variable responses* ___

5. Now **move the telescope one meter to the right** of the straight-on position. Again, **center each object** and **measure how far** it is from the center line.

Object 1: $b2' =$ ___ *Variable responses* ___

Object 2: $b2' =$ ___ *Variable responses* ___

6. Take the **average** of your left and right measurements **for each object** to get your b' value and record it. Hint: Average (b') = $[b1' + b2']/2$

Object 1: $b' =$ ___ *Variable responses* ___

Object 2: $b' =$ ___ *Variable responses* ___



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7. Use the ratio $s'/b' = s/b$ to calculate the distance s from the telescope to each object. Is the distance between the two objects the same as what you get from your direct measurements? (SHOW CALCS AND WRITE OUT EXPLANATIONS!)

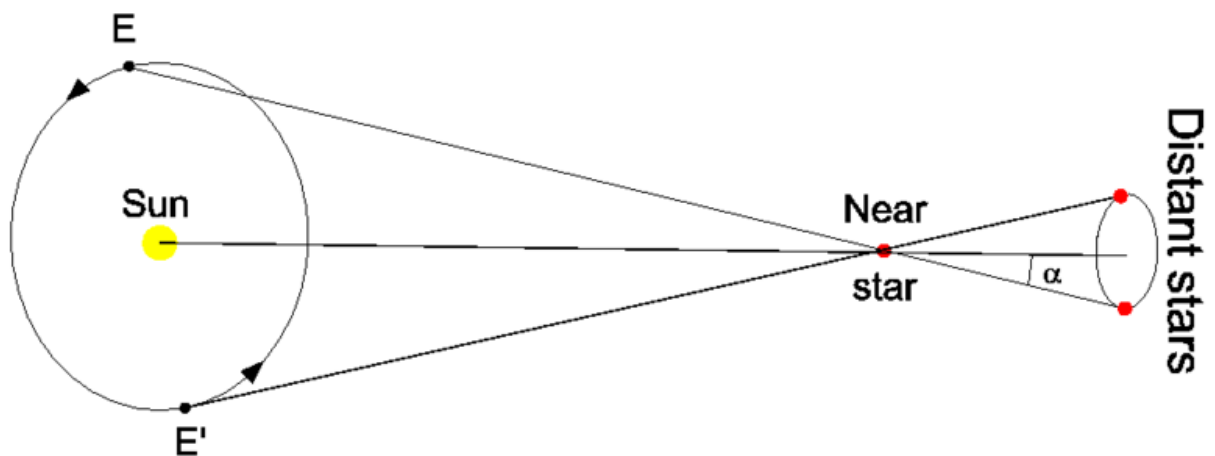
Student responses will vary.

8. Use measuring tape to measure the distance from the telescope (center position) to the different objects. How does the direct measurement compare to your parallax calculations?

Student responses will vary.

9. In your own words, define parallax and draw out a picture to explain it.

Parallax is the angular displacement of an observed object due to changes in the position of the observer. Example picture shown below with parallax labelled.



(Source of image above: commons.wikimedia.org/wiki/File:Annual_parallax.png)

10. How is **parallax related to** the telescope's **distance** to each of the objects?

Inversely proportional; one increases while the other decreases; etc.

11. **Who** would care about **using parallax**? **Why** would this be important for them?

Astronomers, aerospace engineers, space scientists, etc.



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12. Describe any **2 possible sources of error** when you and your group were conducting this experiment, as well as how they may have affected your data/calculations.

Student responses will vary.

Source of Activity and Handout Instructions/Procedures:

www.sciencebuddies.org/science-fair-projects/project-ideas/Astro_p019/astronomy/similar-triangles-using-parallax-to-measure-distance