Creating and Solving Two-Variable Equations

Lesson Plan for Grades: 8th grade
Length of Lesson: 90 minutes

Authored by: Tien Vo for the Environmental Science Institute
Date created: April 24, 2020

Subject area/course: Mathematics

Materials:
- 15-20 Exploration Worksheets
- 15-20 Exit Tickets
- Projector with internet connection

TEKS/SEs:

§111.28. Grade 8, Adopted 2012.

(b) Knowledge and skills.

(8) Expressions, equations, and relationships. The student applies mathematical process standards to use one-variable equations or inequalities in problem situations. The student is expected to:

(A) write one-variable equations or inequalities with variables on both sides that represent problems using rational number coefficients and constants;

(B) write a corresponding real-world problem when given a one-variable equation or inequality with variables on both sides of the equal sign using rational number coefficients and constants;

(C) model and solve one-variable equations with variables on both sides of the equal sign that represent mathematical and real-world problems using rational number coefficients and constants;

(9) Expressions, equations, and relationships. The student applies mathematical process standards to use multiple representations to develop foundational concepts of simultaneous linear equations. The student is expected to identify and verify the values of x and y that simultaneously satisfy two linear equations in the form $y = mx + b$ from the intersections of the graphed equations.

Creating and Solving Two-Variable Equations

Lesson objective(s):
- Students will be able to create two-variable linear equations from real-world problems.
- Students will be able to identify and verify the two variables that satisfy the simultaneous linear equations.
- Students will be able to represent the simultaneous linear equations by graphing and identify the solution to those equations that exist at the intersection of the two graphs.

Differentiation strategies to meet diverse learner needs:
- The lesson plan will include oral, written, and visual explanations to accompany the different learning styles.
- The students can also receive help from each other in discussion and exploration.
- The teacher is available to guide the students through difficult tasks.

ENGAGEMENT (10 minutes)
- Explain to the class that scientists are studying brain-machine interfaces to help people with physical disabilities regain motor skills. The method is transferring information from the brain through wires to a physical machine, which translates the brain activity into a motor command like "move" or "stop."
- As a class watch *Hot Science – Cool Talk* # 122 "Robots Controlled By Your Mind" with Dr. Jose Contreras-Vidal. Play from 21:40 to 22:10 then 25:00 to 26:00.
- Explain to the class that even complex problems can usually generate an equation, which can predict certain trends that are helpful for development of solutions. Scientists can come up with an equation before the experiment and tests for its accuracy, or develop an equation during the experiment, and even after the experiment using all gathered information.
- Explain to the class that today we will be learning how to create and solve systems of equations with 2-variable.
- Watch as a class: Linear Equations in 2 Variables – Word Problem by Don't Memorise

EXPLORATION (20 minutes)
- Divide the students into groups of 4.
- Students will work together in groups to complete the exploration worksheet below to the best of their ability, since there will be a competition during the explanation.

EXPLANATION (30 - 40 minutes)
- Teacher will go over the exploration worksheet by first asking for volunteers from each group for every question.
- If the original group did not get the correct answer or miss some information (they still receive 1 credit for partial answer), other groups have a chance to steal by raising their hand and providing the correct answer with explanation (1 credit).
- After 3 tries, including from the original group, if the students still do not have the correct answer, the teacher will pause the game and go over the problem as a class.
- For section 3, each group will present their created problem in front of the class, and other groups will have 5-7 minutes to solve. The groups that get the correct answer will get a point.

Creating and Solving Two-Variable Equations

ELABORATION (10 minutes)
- The teacher will ask one member from each group to graph one system of equations from each section of the exploration worksheet.
- The teacher will ask the class for a pattern in each graph.
- Students should be able to identify that the solutions for each system of equations lay at the intersection of the graphs of the 2 equations.
- Teacher will summarize what the students learned today and answer any questions students might have.

EVALUATION (10 minutes)
- Evaluation will take place throughout the lesson, including during exploration and explanation as a collaborative effort.
- Students will demonstrate their individual understanding by completing the exit ticket below.

SOURCES AND RESOURCES
- *Hot Science – Cool Talk* #122 Robots Controlled By Your Mind with Dr. Jose Contreras-Vidal, https://youtu.be/TUtPJgr6vl8
- Linear Equations in 2 Variables – Word Problem by Don't Memorise, https://youtu.be/lhQuiC9de98

Creating and Solving Two-Variable Equations

Exploration Worksheet

Student's name: Group's name:

Section 1: Create a system of equations for each word-problem below. Don't solve for the answer.

Problem 1: Jennifer and her friends went to an amusement park last Saturday. The entrance ticket into the park was $5 for each person and the ticket for each ride is $2. The total number of tickets bought was 15 tickets (including entrance and ride tickets). Jennifer and her friends spent a total of $45. How many entrance tickets were bought and how many rides tickets were bought?

Problem 2: Tom decides to sell lemonades for a fundraiser. Customers can buy either the original lemonades for $3 each or the pink lemonade for $4 each. On the first day, Tom sold lemonades and got a total of $70. The amount of original lemonade sold doubled the amount of pink lemonade sold. How many original lemonades were sold? How many pink lemonades were sold?

Problem 3: A basketball team scored a total of 71 points in a competition. They made a total of 30 two-point and three-point baskets. How many two-point shots were made? How many three-point shots were made?

Creating and Solving Two-Variable Equations

Section 2: Solve for the following x and y that would satisfy each system of equations. Verify the answers by substituting x and y into the original equations. Show your steps.

Problem 1: $x + y = 5$

$6y + 3x = 21$

Problem 2: $11 = 5x - 2y$

$x + y = 12$

Problem 3: $3x + y = 2$

$x - 2y = -18$

Section 3: Come up with a word-problem that would generate a system of equations similar to the ones above. Solve and verify the answers. Don't show it to other groups.

Creating and Solving Two-Variable Equations
Exit Ticket

Student's name:

1) Create a system of equations for the following word-problem.

May received a score of 90 on her math test, which consisted of multiple choices and free-response questions. Each multiple choice is 5 points and each free response is 10 points. She got a total of 15 questions correct. How many multiple choices May got correct? How many free-response questions did she get correct?

2) Solve the following system of equations. Show your steps.

$2x + 3y = 7$
$y - 8 = 5x$

3) Graph the following system of equations. Find the answer on the graph and verify the answer by substituting the answer into the equations.

$x = y - 2$
$2x + 2y = 20$

Creating and Solving Two-Variable Equations

Exploration Worksheet [Answers]

Student's name: Group's name:

Section 1: Create a system of equations for each word-problem below. Don't solve for the answer.

Problem 1: Jennifer and her friends went to an amusement park last Saturday. The entrance ticket into the park was $5 for each person and the ticket for each ride is $2. The total number of tickets bought was 15 tickets (including entrance and ride tickets). Jennifer and her friends spent a total of $45. How many entrance tickets were bought and how many rides tickets were bought?

$$5x + 2y = 45$$
$$x + y = 15$$

Problem 2: Tom decides to sell lemonades for a fundraiser. Customers can buy either the original lemonades for $3 each or the pink lemonade for $4 each. On the first day, Tom sold lemonades and got a total of $70. The amount of original lemonade sold doubled the amount of pink lemonade sold. How many original lemonades were sold? How many pink lemonades were sold?

$$3x + 4y = 70$$
$$x = 2y$$

Problem 3: A basketball team scored a total of 71 points in a competition. They made a total of 30 two-point and three-point baskets. How many two-point shots were made? How many three-point shots were made?

$$2x + 3y = 71$$
$$x + y = 30$$

Creating and Solving Two-Variable Equations

Section 2: Solve for the following x and y that would satisfy each system of equations. Verify the answers by substituting x and y into the original equations. Show your steps.

[Answer Key]

Problem 1: $x + y = 5$

$\qquad 6y + 3x = 21$

Work may vary. Possible solution below:

-3(x +y) = -3(5)

6y + 3x = 21

-3x – 3y= -15

3x + 6y = 21

3y = 6

y = 2

x + 2 = 5

x = 3

final answer x = 3, y = 2

Substituting in first equation:

x + y = 5

3 + 2 = 5

5 = 5

Substituting in second equation:

6y + 3x = 21

6 (2) + 3 (3) = 21

12 + 9 = 21

21 = 21

Creating and Solving Two-Variable Equations

Problem 2: $11 = 5x - 2y$

$x + y = 12$

Work shown may vary. Possible solution below:

$5x - 2y = 11$
$2 (x + y) = 2 (12)$

$5x - 2y = 11$
$2x + 2y = 24$

$7x = 35$

$x = 5$

$x + y = 12$
$5 + y = 12$
$y = 7$

final answer $x = 5, y = 7$

Substituting in first equation:
$5x - 2y = 11$
$5(5) - 2 (7) = 11$
$25 - 14 = 11$
$11 = 11$

Substituting in second equation:
$x + y = 12$
$5 + 7 = 12$
$12 = 12$

Creating and Solving Two-Variable Equations

Problem 3: $3x + y = 2$

$\qquad x - 2y = -18$

Work shown may vary. Possible solution below:

$x - 2y = -18$
$2(3x + y) = (2)2$

$x - 2y = -18$
$6x + 2y = 4$

$7x = -14$

$x = -2$

$3x + y = 2$
$3(-2) + y = 2$
$-6 + y = 2$
$y = 8$

final answer $x = -2$, $y = 8$

Substituting in first equation:
$x - 2y = -18$
$(-2) - 2(8) = -18$
$-2 - 16 = -18$
$-18 = -18$

Substituting in second equation:
$3x + y = 2$
$3(-2) + 8 = 2$
$-6 + 8 = 2$
$2 = 2$

Section 3: Come up with a word-problem that would generate a system of equations similar to the ones above. Solve and verify the answers. Don't show it to other groups.

Answers will vary.

Creating and Solving Two-Variable Equations

Exit Ticket [Answer Key]

Student's name:

1) Create a system of equations for the following word-problem.

May received a score of 90 on her math test, which consisted of multiple choices and free-response questions. Each multiple choice is 5 points and each free response is 10 points. She got a total of 15 questions correct. How many multiple choices May got correct? How many free-response questions did she get correct?

Work shown may vary.

x: multiple choice questions
y: free-response questions

$$5x + 10y = 90$$
$$x + y = 15$$

$$5x + 10y = 90$$
$$-5 (x + y) = -5 (15)$$

$$5x + 10y = 90$$
$$-5x - 5y = -75$$

$$5y = 15$$

$$y = 3$$

$$x + y = 15$$
$$x + 3 = 15$$
$$x = 12$$

final answer x= 12; y = 3
May had 12 multiple choice questions and 3 free-response questions correct

$$5x + 10y = 90 \text{ (substituting in first equation)}$$
$$5 (12) + 10 (3) = 90$$
$$60 + 30 = 90$$
$$90 = 90$$
$$x + y = 15$$
$$3 + 12 = 15$$
$$15 = 15$$

Creating and Solving Two-Variable Equations

2) Solve the following system of equations. Show your steps.

$2x + 3y = 7$
$y - 8 = 5x$

Work shown may vary.

$2x + 3y = 7$
$y - 5x = 8$

$2x + 3y = 7$
$-3(y - 5x) = -3(8)$

$2x + 3y = 7$
$15x - 3y = -24$

$17x = -17$

$x = -1$

$y - 5x = 8$
$y = 5x + 8$
$y = 5(-1) + 8$
$y = -5 + 8$
$y = 3$

answer: $x = -1$, $y = 3$

$2x + 3y = 7$ (substituting in first equation)
$2(-1) + 3(3) = 7$
$-2 + 9 = 7$
$7 = 7$

$y - 5x = 8$ (substituting in second equation)
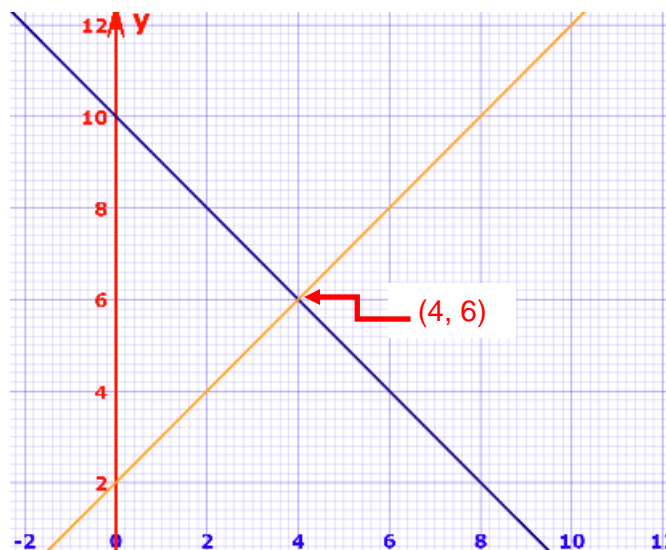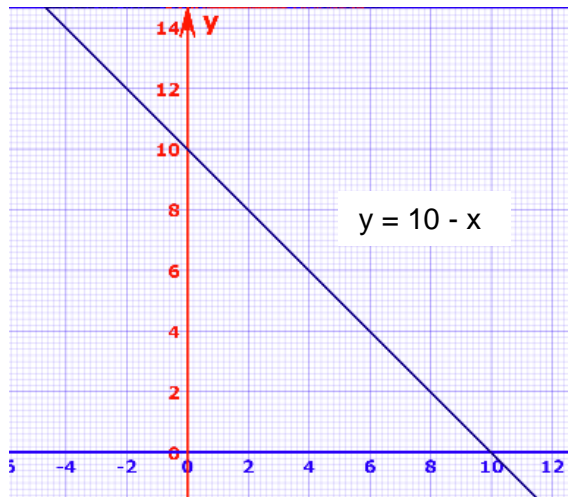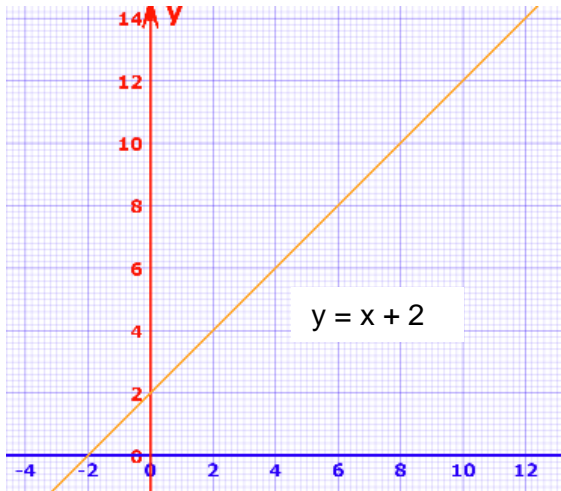$3 - 5(-1) = 8$
$3 + 5 = 8$
$8 = 8$

Creating and Solving Two-Variable Equations

3) Graph the following system of equations. Find the answer on the graph and verify the answer by substituting the answer into the equations.

x = y - 2
2x + 2 y = 20

Rearranging the equations to y = mx + b

y = x + 2
y = 10 – x

y = x + 2

y = 10 - x

(4, 6)

Creating and Solving Two-Variable Equations

Substituting answer (4,6) in original equations

$$x = y - 2$$
$$2x + 2y = 20$$

$$x = y - 2$$
$$4 = 6 - 2$$
$$4 = 4$$

$$2x + 2y = 20$$
$$2(4) + 2(6) = 20$$
$$8 + 12 = 20$$
$$20 = 20$$