

Treasure Hunt: GPS and Trilateration

Lesson Plan for Grades __9-12__

Length of Lesson: _1_ hr _0_ min

Authored by: __Maisha Rumman__ for UT Environmental Science Institute

Date created: [11/14/2014]

Subject area/course: Geometry

Materials:

- graph paper
- geometry compass
- laptop or computer

TEKS/SEs:

§111.34. Geometry (One Credit).

(2) Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures. The student is expected to:

- use constructions to explore attributes of geometric figures and to make conjectures about geometric relationships
- make conjectures about angles, lines, polygons, circles, and three-dimensional figures and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.

(4) Geometric structure. The student uses a variety of representations to describe geometric relationships and solve problems. The student is expected to select an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

(7) Dimensionality and the geometry of location. The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. The student is expected to:

- Use one- and two-dimensional coordinate systems to represent points, lines, rays, line segments, and figures.

§111.39. Algebra I, Adopted 2012 (One Credit).

(5) Linear functions, equations, and inequalities. The student applies the mathematical process standards to solve, with and without technology, linear equations and evaluate the reasonableness of their solutions. The student is expected to:

- (C) solve systems of two linear equations with two variables for mathematical and real-world problems.

Lesson objective(s):

Students will learn about how the GPS works and about trilateration.

Treasure Hunt: GPS and Trilateration

Differentiation strategies to meet diverse learner needs:

Learners learning in a second language

- Think pair share: this strategy is when students are asked to pair up to think about the answer to a question or brainstorm activity. Then, they are asked to share their thoughts with the class.
- Extend wait time: teachers should wait at least three seconds after posing a question in order to let students process their thoughts.
- Review and practice concepts—have students teach each other concepts and note similar and different things that they did.
- Implement more formative assessment. Formative assessment is a strategy teachers use to change their lesson plans on the fly. For example, if a teacher poses a question that students are already supposed to know for a lesson but the teacher finds out that students don't actually know the answer—the teacher will then modify the lesson plan to include a mini lesson on that question or concept.
- Give students an opportunity to practice speaking (before presentation)
 - have them record themselves and send it to teacher for corrections

Students with reading and writing difficulties

- Give students plenty of space to write
- If students have visual impairments, provide recordings or text that is enlarged

Students with attention and behavioral challenges

- Use open ended questions that allow students to express their thoughts
- Structure long term assignments into smaller and manageable segments. Have students work on one task at a time.
- Build in mental breaks that allow students to regroup, stretch, and get out of their seats
- Have students keep an organizer or a binder where they can keep all their work materials in the order that they received them

ENGAGEMENT

Most of you have cell phones that have GPS and you've most likely watched your parents use it to get to a location. How do you think a GPS actually works? What kind of information is it getting in order to figure out your location?

Students will be asked to get in pairs and discuss these questions. Teacher will specify to students that they have two minutes to brainstorm on their own and then discuss their thoughts with a partner. At this time, the teacher should be circling the class and asking partners questions such as "how do you know?" or "what led you to this idea" in order to probe students to further elaborate on their ideas. After giving the partners enough time (approximately five to seven minutes) to discuss the answers, teacher will then call on students randomly to present their ideas.

Teacher will show them a short clip of this intro video (end/pause video at 1:06):

<https://www.youtube.com/watch?v=chNQW22vVNI>

Teacher will tell students that they will be exploring the math behind GPS in the lesson.

Treasure Hunt: GPS and Trilateration

EXPLORATION

Students will be placed in groups and asked to find the treasure.

Groups will be given graph paper, the clues to the location of the 3 satellites (in coordinates), a drawing compass tool, and a few hints of where the treasure is in relation to the satellites. Using these materials, they will be asked to determine the location of the treasure.

Students will be asked to complete a worksheet for this activity where students will describe how they found the location of the treasure.

Teacher will then ask students to work individually in order to answer the last question on the worksheet. For the last question, students will find the two intersections of two circle equations of two satellites using systems of equations. After they are done, they will pair up with two other people who have the two points of intersection of the two other combinations of satellites (remember there are three satellites in total) and figure out where the treasure is. The treasure will be the coordinate point that is similar for all three students.

EXPLANATION

Groups will present on how they found their treasure both using the geometric compass and using systems of equations. They will also compare how their strategies were different or the same.

Teacher will ask a few students, randomly, to present their solutions to how they would solve the problem with just equations.

Teacher will show a video of how the GPS actually works and introduce the concept of three dimensionality as well as how it's related to the two dimensional concept that students worked with earlier.

Teacher will show this video (start at 0:14 and pause/end video at 3:07)

Video: <https://www.youtube.com/watch?v=0n0T992ccik>

Teacher can pause the video at certain points to ask questions and introduce discussion.

ELABORATION

How can GPS be applied usefully? What kinds of questions can it help us answer and what kinds of problems can it help us solve?

Students will be asked to refer to this website to answer these questions (they can figure out useful applications of GPS by clicking on the "Applications" tab at the top of the page) : <http://www.gps.gov/students/>

Challenge for advanced students: how would introducing three dimensionality change your equations? How would introducing more than three satellites change your equations?



Treasure Hunt: GPS and Trilateration

EVALUATION

Students will be evaluated on their presentation and the solution they came up with to find their treasure.

SOURCES AND RESOURCES

- Dr. Todd Humphrey's Hot Science – Cool Talks Lecture # 93

Resources for teachers and students:

intro video

<https://www.youtube.com/watch?v=chNQW22vVNI>

Great sites:

<http://timeandnavigation.si.edu/satellite-navigation>

<http://www.gps.gov/students/>

<https://www.youtube.com/watch?v=4O3ZVHVFhes>

http://oceanservice.noaa.gov/education/lessons/where_you_are.html

http://oceanservice.noaa.gov/education/lessons/meet_geodesy.html (lesson plan idea)

Great powerpoint about GPS:

http://web.gps.caltech.edu/classes/ge167/file/GPS_Lecture_1.pptx

Videos from gps.gov:

<http://www.gps.gov/multimedia/videos/>

Plot points and find intersections:

<https://www.desmos.com/calculator>

Plug in numbers to the quadratic equation:

<http://www.math.com/students/calculators/source/quadratic.htm>

Treasure Hunt: GPS and Trilateration

EXPLORATION ACTIVITY or ACTIVITIES

Purpose

Students will explore the mathematics behind GPS.

Materials

Graph paper
Ball bearing/geometric compass

Safety Information

Students should be careful if using geometric compasses with sharp points. However, they do not need to worry if they are working with a safety compass.

Procedure

1. Students will be divided into groups of no more than 3.
2. Groups will be given graph paper, geometric compass, and clues to where their satellites are (in coordinates) and also how far they are from the treasure.
3. Students in each group will form a plan about how to find the treasure and record it on their worksheet
4. They will then find the treasure using the tools they were given
5. Lastly students will individually work on the last question on the worksheet. For the last question, students will be asked to work individually and use systems of equations and the circle equations of 2 satellites to find the two points of intersection of the two circles.
6. Teacher will number off students and will number them either 1, 2, or 3.
7. Students assigned number 1 will work on equations 1 and 2. Students assigned number 2 will work on equations 2 and 3. And students assigned number 3 will work on equations 1 and 3.
8. Teacher will show students how to use the calculator on a website in order to graph equations in order to confirm that their calculations for the systems of equations are correct: <https://www.desmos.com/calculator>
9. Students will then pair up with two other students who have two different combinations of satellite systems of equations and figure out where the treasure is.
10. Students in each group will then present on how they found their treasure both using the geometry compass and systems of equations.

Treasure Hunt: GPS and Trilateration

TEACHER PAGE(S)

Treasure Hunt Worksheet

Instructions: In this activity, you're responsible for finding treasure given some clues about where it is

- 1) Gather materials: Graph paper and compass
- 2) Use these clues to figure out where your treasure is (make sure to use your graph paper and compass)
 - a. Clue 1: Your treasure is 9 units away from the satellite at coordinate (0, 0)
 - b. Clue 2: It is $3\sqrt{2}$ units way from the satellite at coordinate (12, 3)
 - c. Clue 3: And it is 12 units away from the satellite at coordinate (9, -12)
- 3) Talk amongst your group and figure out how you will use these clues, graph paper and the compass to find the treasure and then record your plan and answer below. Make sure to attach your graph to this page.

Plan

In the box below, thoroughly describe the plan that your group will use in order to find your treasure. In other words, explain what method you used to find it. Attach your graph to this sheet and show your work.

We used the compass and drew a circle around the satellite using the distance of the satellite from the treasure. We did this for the two other satellites as well. The point of intersection of the circle surrounding the three satellites is where the treasure is located.

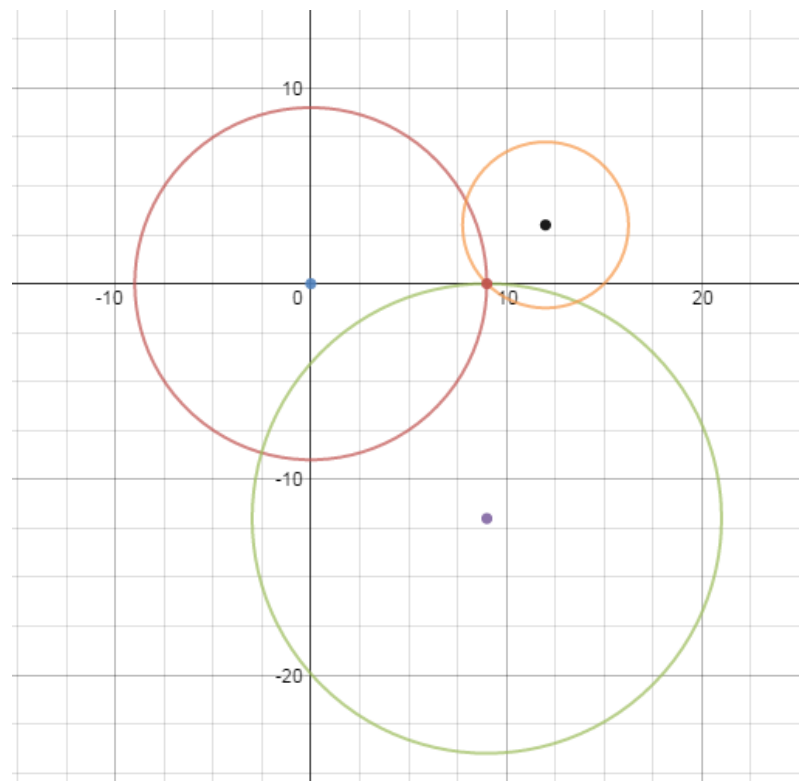
Treasure Hunt: GPS and Trilateration

TEACHER PAGE(S)

Graph for first question of the worksheet

Note: students finished product should look like the graph below.

1	$x^2 + y^2 = 81$	✕
2	(0,0)	✕
3	$(x-9)^2 + (y+12)^2 = 144$	✕
4	(9,-12)	✕
5	$(x-12)^2 + (y-3)^2 = 18$	✕
6	(12,3)	✕
7	(9,0)	✕

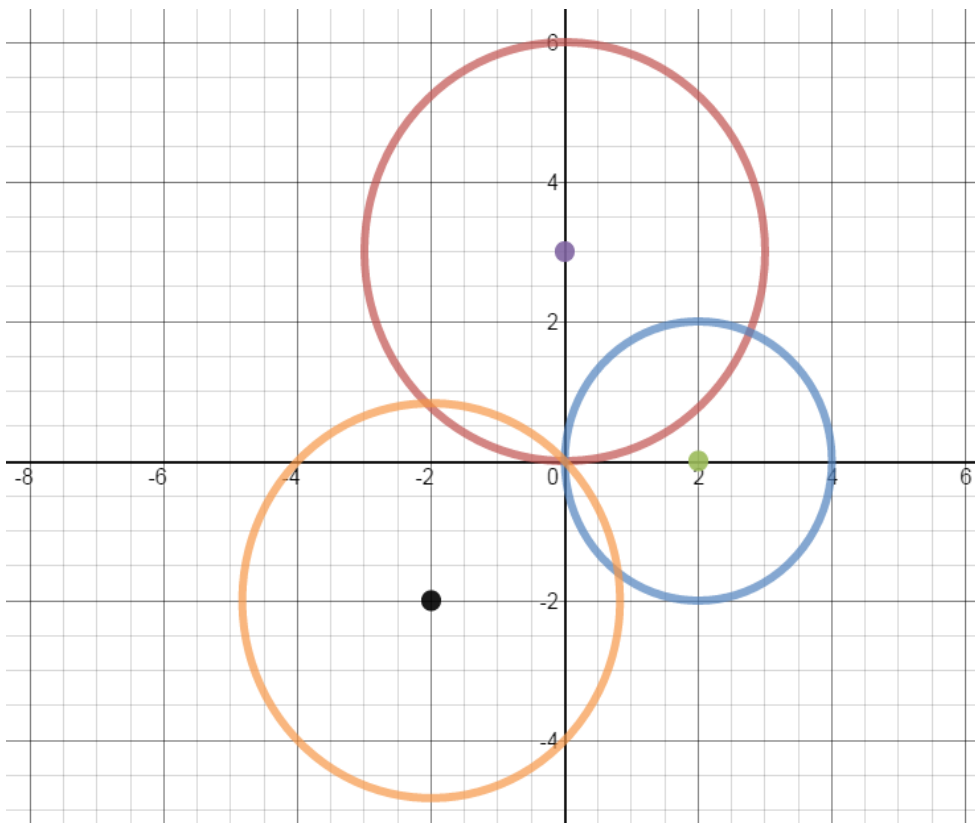


Where is your treasure? (exact coordinate) (9 , 0)

Treasure Hunt: GPS and Trilateration

TEACHER PAGE(S)

Graph for last question



Untitled Graph

1	$(x)^2 + (y - 3)^2 = 9$	✕
2	(0,3)	✕
3	$(x - 2)^2 + (y)^2 = 4$	✕
4	(2,0)	✕
5	$(x + 2)^2 + (y + 2)^2 = 8$	✕
6	(-2, -2)	✕
7	(0,0)	✕
8		

Treasure Hunt: GPS and Trilateration

TEACHER PAGE(S)

Challenge Question: Let's say you weren't given the compass and graph paper. Could you use systems of equations to calculate the coordinates (location) of your treasure given the circle equation of your satellites (ie. the location of the satellite and its distance from the treasure)? To test this, you will be assigned two of the three satellite equations below and use systems of equations to figure out the two points of intersection of the two equations. Then pair up with two students who were assigned two different satellite combinations and figure out where the treasure is.

Equation of satellite 1: $x^2 + (y-3)^2 = 9$

Equation of satellite 2: $(x-2)^2 + y^2 = 4$

Equation of satellite 3: $(x+2)^2 + (y+2)^2 = 8$

Systems of equation for satellite 1 and satellite 2

Step 1)

$$x^2 + (y-3)^2 = 9 \quad x^2 + y^2 - 6y + 9 = 9$$

$$-[(x-2)^2 + y^2 = 4] \rightarrow -x^2 + 4x - 4 - y^2 = -4 \rightarrow 4x = 6y \rightarrow y = 2x/3$$

Step 2)

Students can either manually plug in $y = 2x/3$ into one of the original equations above or they may plug it in the y value (or x value if that's what they choose) using the online desmos graphing tool which will be much faster for them \rightarrow <https://www.desmos.com/calculator>

In the desmos calculator, type in one of the original equations in the blank line, substitute $y = 2x/3$ for the y value, and then attain the two x values

Step 3) Once you have your x values, which in this case it is $x = 0$ and 2.769 , plug in the first value into both of the original equations and then pick the y value that is the same for both of the equations. In this case, it is 0 and 1.846 respectively

Step 4) have students check their answer by graphing the two original satellite equations on desmos and looking at their points of intersection

Treasure Hunt: GPS and Trilateration

TEACHER PAGE(S)

Systems of equation for satellite 2 and satellite 3

Step 1)

$$(x+2)^2 + (y+2)^2 = 8 \quad x^2 + 4x + 4 + y^2 + 4y + 4 = 8$$

$$-[(x-2)^2 + y^2 = 4] \rightarrow -x^2 + 4x - 4 - y^2 = -4 \quad \rightarrow 8x = -4y \rightarrow y = -2x$$

Step 2)

Students can either manually plug in $y = -2x$ into one of the original equations above or they may plug it in the y value (or x value if that's what they choose) using [desmos.com](https://www.desmos.com)

In the desmos calculator, type in one of the original equations in the blank line, substitute $y = -2x$ for the y value, and then attain the two x values

Step 3) Once you have your x values, which in this case it is $x = 0$ and 0.8 , plug in the first value into both of the original equations and then pick the y value that is the same for both of the equations. In this case, it is 0 and -1.6 respectively

Steps 4) Have students check their answer by graphing the two original satellite equations on desmos and looking at their points of intersection

Systems of equation for satellite 1 and satellite 3

Step 1)

$$x^2 + (y-3)^2 = 9 \quad x^2 + y^2 - 6y + 9 = 9$$

$$-[(x+2)^2 + (y+2)^2 = 8] \rightarrow -x^2 - 4x - 4 - y^2 - 4y - 4 = -8 \quad \rightarrow -4x = 10y \rightarrow y = -4x/10$$

Step 2)

Students can either manually plug in $y = -4x/10$ into one of the original equations above or they may plug it in the y value (or x value if that's what they choose) using [desmos.com](https://www.desmos.com)

In the desmos calculator, type in one of the original equations in the blank line, substitute $y = -4x/10$ for the y value, and then attain the two x values

Step 3) Once you have your x values, which in this case it is $x = 0$ and -2.0689 , plug in the first value into both of the original equations and then pick the y value that is the same for both of the equations. In this case, it is 0 and $.828$ respectively

Steps 4) Have students check their answer by graphing the two original satellite equations on desmos and looking at their points of intersection

Treasure Hunt: GPS and Trilateration

STUDENT PAGE(S)

Treasure Hunt Worksheet

Instructions: In this activity, you're responsible for finding treasure given some clues about where it is

- 1) Gather materials: Graph paper and compass
- 2) Use these clues to figure out where your treasure is (make sure to use your graph paper and compass)
 - a. Clue 1: Your treasure is 9 units away from the satellite at coordinate (0, 0)
 - b. Clue 2: It is $3\sqrt{2}$ units way from the satellite at coordinate (12, 3)
 - c. Clue 3: And it is 12 units away from the satellite at coordinate (9, -12)
- 3) Talk amongst your group and figure out how you will use these clues, graph paper and the compass to find the treasure and then record your plan and answer below. Make sure to attach your graph to this page.

Plan

In the box below, thoroughly describe the plan that your group will use in order to find your treasure. In other words, explain what method you used to find it.

Where is your treasure? (exact coordinate) __ (____, ____) __

Challenge Question: Let's say you weren't given the compass and graph paper. Could you use systems of equations to calculate the coordinates (location) of your treasure given the circle equation of your satellites (ie. the location of the satellite and its distance from the treasure)? To test this, you will be assigned two of the three satellite equations below and use systems of equations to figure out the two points of intersection of the two equations. Then pair up with two students who were assigned two different satellite combinations and figure out where the treasure is.

Equation of satellite 1: $x^2 + (y-3)^2 = 9$

Equation of satellite 2: $(x-2)^2 + y^2 = 4$

Equation of satellite 3: $(x+2)^2 + (y+2)^2 = 8$

Treasure Hunt: GPS and Trilateration

STUDENT PAGE(S)