Title: Direction and Displacement

Subject: Mathematics

Grade Level: 10^{th} – 12^{th}

Rational or Purpose: This activity will explore students' knowledge on directionality and displacement. With the use angle and distance, students will learn and practice the use of the law of sine and cosine. Students will see how the Pythagorean Theorem relates to the law of cosine.

Materials:
- Ruler

Lesson Duration: 45 minutes

TEKS Objectives:
§111.35. Precalculus
(c) Knowledge and skills:
(P.3) The student uses functions and their properties, tools and technology, to model and solve meaningful problems.
(E) solve problems from physical situations using trigonometry, including the use of Law of Sines, Law of Cosines, and area formulas and incorporate radian measure where needed.

Background Information: To understand sea turtles' navigation, scientists attach Global Positioning System (GPS) instruments on the turtles so that scientists can locate the actual position of turtles. When turtles travel between continents, they do not travel in a straight line; they make turns along their journey.

If scientists need to find a specific turtle, it would be best to travel the shortest distance, instead of following turtle's path. In science, the shortest distance between two locations is called "displacement." The actual path traveled is called "distance." An analogy to describe these two terms is running on a track. A runner finishes running 400 meters and returns to the starting point. Their running distance is 400 meters, but the running displacement is 0 meters.

Activity: Students have already used the function of sine, cosine, and tangent to calculate angles and/or the length of a side. Such calculations are restricted to right triangles only. The law of sine and cosine can help determine the angle and length of *any* triangles.

This activity also introduces two ways to describe directionality: compass bearing and true bearing. These methods will help students identify locations if they know the given angle and distance.

Procedure: Follow the teacher handout.

Direction and Displacement: Teacher's Guide

Name: _____ Date: _____

Class Section: _____

Compass bearing and *true bearing* describe directionality. You will want to review the information below before going into the activity.

All directionality in *compass bearings* are measured either from the north or from the south (but never from the east or from the west). Then, it expresses the angle from north or south to east or west. Some simple examples are:

N45ºE N60ºW S30ºE S15ºW

The angle in the middle must be between 0º and 90º.

True bearings measure directionality from the north in a clockwise direction. The bearing is expressed with a 3-digit angle that ranges from (or equal to) 0º to any angle less than 360º.

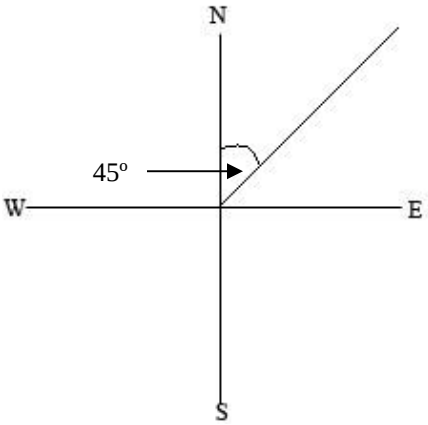
Students may find it difficult to define the exact value of an included angle. Remember that each quadrant is 90º. Also remember angle properties with parallel lines (included angles with two parallel lines are equal).

The Law of Sine: $a / \sin A = b / \sin B = c / \sin C$

The Law of Cosine: $c^2 = a^2 + b^2 - 2ab \cos C$

Both laws allow us to use trigonometric functions to calculate the sides and/or angles desired. These two laws are not restricted to right-angled triangles. If you apply the Law of Cosine with a right-angled triangle, you may find something special. It will be mentioned later in this activity.

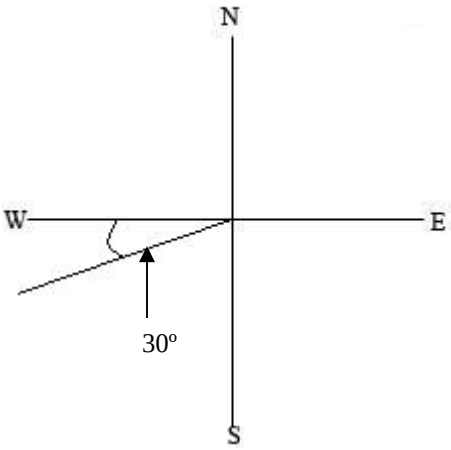
Example



Compass Bearing: North 45º East (N45ºE)

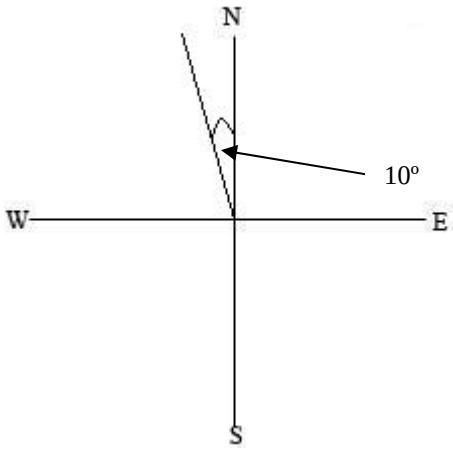
True Bearing: 045º

Practice Exercise



Compass Bearing: S60ºW

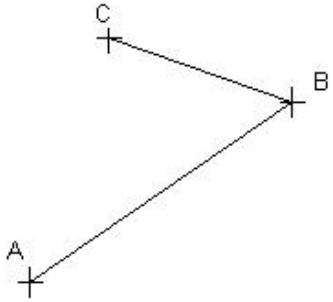
True Bearing: 240º



Compass Bearing: N10ºW

True Bearing: 350º

<u>Activity: Where does the turtle go?</u>



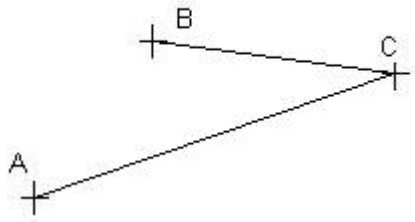
Turtle A initially takes a path from point A to point B in a direction of 050º. After it arrives at point B, it changes its direction to N60ºE, traveling to point C.

The distance AB is 20km.
The distance BC is 15km.

What is the shortest distance (or displacement) between A and C?

Find angle ABC, then use the Law of Cosine.

By using the Law of Cosine, what is the value of angle CAB?



Turtle B walks 12 miles from point A to point C in a direction of N55ºE. Then, the turtle changes its direction at point C. It then moves to a direction of 295º from point C. After 9 miles, it reaches point B.

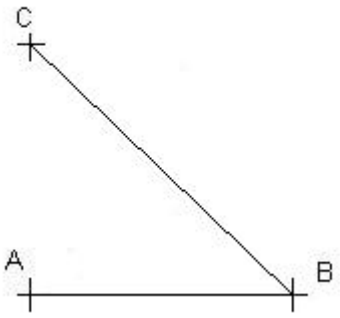
What is the displacement between point A and B?

Find Angle ACB, then use the Law of Cosine.

How many degrees is angle ABC? How do you find it?

Students can use either the Law of Sine or Cosine to find angle ABC.

Consider the following path:



Turtle C walks 15 miles east from point A to reach point B. Then, it begins to walk in a direction of 310º. It stops after walking a distance of 25 miles.

How many degrees is angle ABC?

Find the displacement between point A and C.

Use the Law of Cosine. ΔABC turns out to be a right-angled triangle.

Using the Law of Cosine, find the size of angle BAC.

90º

Based on the answer you have just provided, is there another method to find the displacement between point A and C? What is it?

Since it is a right-angled triangle, students can use any trigonometric function and Pythagorean Theorem to solve the displacement AC. ***It is important to show how the Pythagorean Theorem is related to the Law of Cosine.*** *("2ab cos C" becomes zero when C=90º).*

Direction and Displacement

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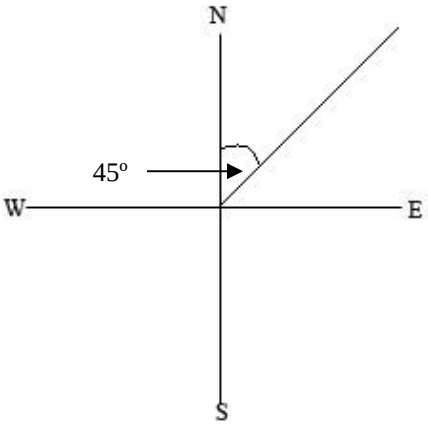

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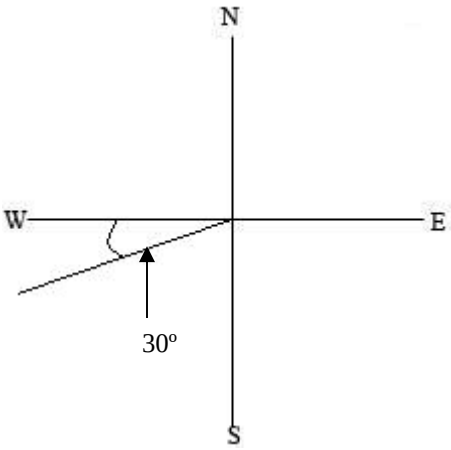
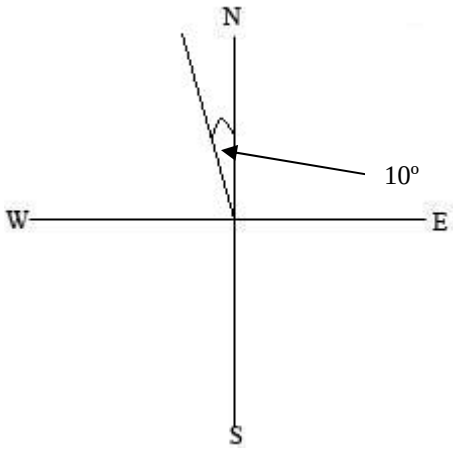


Compass Bearing:

True Bearing:



Compass Bearing:

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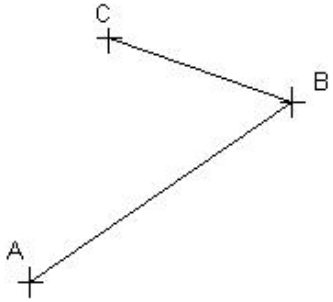

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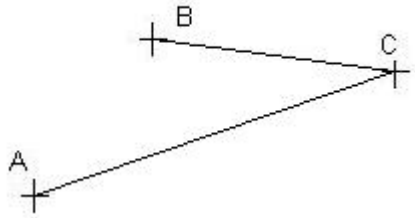

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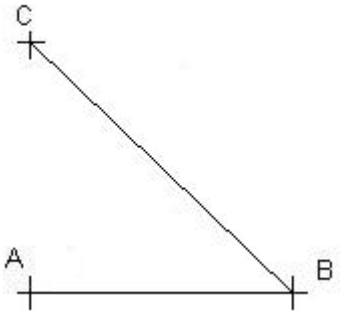

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