Whale Disappearance: A Slippery Slope

Subject: Math

Grade Level: Calculus AB/BC

Rational or Purpose: Students will be introduced to the concept of slope and how to use a tangent line to make short-term predictions. This activity relates mathematics to real life situations and will raise students' awareness of how whaling affects whale populations.

Materials:
- Graph paper
- Ruler

Lesson Duration: 45 minutes

Source of Lesson: *Hot Science – Cool Talks* CD-ROM # 45: "The History and Future of Whales"

Topics covered for Calculus AB and BC:

Concept of the derivative:
- Derivative presented graphically, numerically, and analytically
- Derivative interpreted as an instantaneous rate of change
- Derivative defined as the limit of the different quotient
- Relationship between differentiability and continuity

Derivative at a point
- Slope of a curve at a point
- Tangent line to a curve at a point and local linear approximation
- Instantaneous rate of change as the limit of average rate of change
- Approximate rate of change from graphs and tables of values

Activity:

Engage the students by watching a microdocumentary on *The Secret Lives of Whales* (www.stanford.edu/group/Palumbi/microdocs.html#secret). Then, provide background information about Fin Whale catch records and how to calculate slope. Next, have students plot the data on a graph paper and answer questions.

Background Information:

The international whaling industry grew rapidly after the creation of factory vessels in 1926[1]. In 1931, the number of catches climbed to an historical high. During 1950s and 1960s, catches of Fin Whale were *constantly* kept at high quantity.

The number of catches is inversely related to the number whales. In other words, as the number of whales caught increases, the population of whales in the oceans decreases. Often, the decrease of population follows a trend. In order to recognize the trend of the population, it is very important to understand the concepts of slope.

Slope is used to measure the steepness of an incline or decline of a straight line. The method to measure a slope of a straight line is to divide the change of *y*-value by the change of *x*-value. (If there are only points on a graph, link two desired points with a straight line and follow the same procedure.) Slope is usually represented by alphabet letter *m*. A positive slope represents an incline of the line, and a negative slope represents a decline.

It is important to know that every point on a *straight line* has the same slope. The absolute value of the slope reflects the line's steepness (the greater of the absolute value of a slope, the greater steepness of a line). Slope is sometimes represented by terms such as rate of change, speed, velocity, or other terms that reflect a change of dependent variable in respect to an independent variable.

Linear approximation is a method to estimate future quantities, such as population. The "linear" object that is used to estimate is a tangent line. By projecting a tangent line from a particular point of the graph, it is possible to predict a probable future quantity based on recent trends (assuming that those trends continue). A major limitation of linear approximation is that many real life situations do not follow a linear pattern. Therefore, it is generally better to use linear approximation on for short-term (rather than long-term) prediction.

Procedure:

Presented here is a set of data that describes the number of Fin Whales caught between 1925 and 1979 (Figure 1). Analyze the data and represent it graphically. Name the graph with a suitable title, and set up two axes with proper labels. Plot each point on your graph paper and connect the points with straight lines.

[1] Clark, CW and R. Lamberson. 1982. An economic history and analysis of pelagic whaling. *Marine Policy* 6 (2): 103-120.

Year	# of Fin Whales	Year	# of Fin Whales
1925	2500	1951	17500
1927	4000	1953	22000
1929	3500	1955	26000
1931	8500	1957	26000
*1932	1000	1959	26000
1933	4500	1961	27500
1935	11500	1963	24000
1937	14500	1965	13000
*1938	26000	1967	3500
1939	19000	1969	3500
1941	7500	1971	3500
1943	500	1973	2500
1945	1000	1975	2000
1947	13000	1977	200
1949	17500	1979	200

Figure 1. The number of Fin Whales caught between 1925 and 1979.
(*) indicates the years with important information that are not included in the regular two-year interval.

Questions: (Show your work for all calculations)

1. What is slope between 1933 and 1943?

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2. On the same graph paper, draw a tangent line at t=1933. What is the equation of the tangent line?

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3. Using the tangent line, estimate the number of catches when t=1938. Is it a good estimate?

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4. Can you use the same tangent line to estimate the number of catches when t =1945? Give a reasonable explanation for your answer. (Hint: what major event happened during the 1940s?)

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5. What is/are the advantage(s) and disadvantage(s) of linear approximation in terms of accuracy of the estimation?

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6. What do you predict will happen to the Fin Whale population and number caught in the future? Why?

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Additional Facts:

From 1943 to 1945, almost all sailing in open oceans was stopped because of World War II. Countries required whaling industries to provide their fleets for military use, so only few ships could travel and whale. Although World War II temporarily caused a reduction in the number of whales caught, the whaling industries were able to recover shortly after the end of war.

In 1961, the population of Fin Whale began to decrease dramatically, which made catching Fin Whales more difficult. This is reflected on the table by the significant drop in the number of Fin Whales caught after 1961.