Environmental Science Institute
The University of Texas at Austin

To Infinity and Beyond
Dr. Michael Starbird

This file contains Middle and High School Texas Essential Knowledge and Skills for Mathematics referencing Professor Starbird's chapter on Infinity (highlighted in red).

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Chapter 111. Texas Essential Knowledge and Skills for Mathematics
Subchapter B. Middle School (grades 6-8)
Statutory Authority: The provisions of this Subchapter B issued under the Texas Education Code, §28.002, unless otherwise noted.

§111.21. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 6-8.
The provisions of this subchapter shall be implemented by school districts beginning September 1, 1998, and at that time shall supersede §75.27(g) and §75.43(a) and (b) of this title (relating to Mathematics).
Source: The provisions of this §111.21 adopted to be effective September 1, 1998, 22 TexReg 7623.

(b) **Knowledge and skills.**

(6.1) **Number, operation, and quantitative reasoning.** The student represents and uses
rational numbers in a variety of equivalent forms.
Sec 3.2: Comparing the Infinite, "Rational Numbers," pp.152-155.

(6.2) **Number, operation, and quantitative reasoning.** The
student adds, subtracts, multiplies, and divides to solve problems and justify solutions.
The student is expected to:
(A) model addition and subtraction situations involving fractions with objects, pictures, words, and numbers;
Sec 3.1: Beyond Numbers, “What Does ‘2’ Mean?” p. 139.
Sec 3.1: Beyond Numbers, “Correspondence” p. 140.
Sec 3.1: Beyond numbers, “Mindscapes” (activities) pp. 141-144.
(B) use addition and subtraction to solve problems involving fractions and decimals;
Sec. 3.3: The Missing Number, "Unequal Decimals," p.164.

(6.3) **Patterns, relationships, and algebraic thinking.** The student solves problems involving proportional relationships. The student is expected to:
(B) represent ratios and percents with concrete models, fractions, and decimals;
Sec. 3.3: The Missing Number, "Unequal Decimals," p.164.

(6.6) **Geometry and spatial reasoning.** The student uses geometric vocabulary to describe angles, polygons, and circles. The student is expected to:
Sec. 3.5: Straightening Up the Circle, pp. 190-198.

(6.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to identify location in two dimensions. The student is expected to locate and name points on a coordinate plane using ordered pairs of non-negative rational numbers.
Sec 3.2: Comparing the Infinite, pp.152-155.

(6.8) **Measurement.** The student solves application problems involving estimation and measurement of length, area, time, temperature, capacity, weight, and angles. The student is expected to:
(A) estimate measurements and evaluate reasonableness of results;
Sec 3.4: Travels Toward the Stratosphere of Infinities, “Mindscape” (activities), pp. 185-189.

(6.11) **Underlying processes and mathematical tools.** The student applies Grade 6 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:
(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table,working a simpler problem, or working backwards to solve a problem; and
(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
(A) – (D):
Sec 3.1: Beyond Numbers, "Mindscapes” (activities) p. 141-144.
Sec 3.2: Comparing the Infinite, “Mindscapes” (activities) pp. 156-161.
Sec. 3.3: The Missing Number, “Mindscapes” (activities) pp. 168-172.
Sec 3.4: Travels Toward the Stratosphere of Infinities, “Mindscapes” (activities), pp. 185-189.
Sec. 3.5: Straightening Up the Circle, “Mindscapes” (activities) pp. 201-205.

(6.13) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:
(A) make conjectures from patterns or sets of examples and nonexamples;
Sec. 3.4: Travels Toward the Stratosphere of Infinities, "Sets Within Sets," pp.175-179.
(B) validate his/her conclusions using mathematical properties and relationships.
Sec 3.1: Beyond Numbers, "Mindscapes: Creating New Ideas," p. 143.

Source: The provisions of this §111.22 adopted to be effective September 1, 1998, 22 TexReg 7623. §111.23. Mathematics, Grade 7. September 1997 Original Page B-5

§111.23. Mathematics, Grade 7.

(b) **Knowledge and skills.**

(7.1) **Number, operation, and quantitative reasoning.** The student represents and uses
numbers in a variety of equivalent forms. The student is expected to:
(A) compare and order integers and positive rational numbers;
Sec 3.2: Comparing the Infinite, "Rational Numbers," pp.152-155.
Sec 3.2: Comparing the Infinite, "Integers Equal Naturals," pp. 151-152.
(B) convert between fractions, decimals, whole numbers, and percents mentally, on paper, or with a calculator;
Sec. 3.3: The Missing Number, "Unequal Decimals," p.164.

(7.2) **Number, operation, and quantitative reasoning.** The
student adds, subtracts, multiplies, or divides to solve problems and justify solutions.
The student is expected to:
(A) represent multiplication and division situations involving fractions and decimals with concrete models, pictures, words, and numbers;
Sec 3.1: Beyond Numbers, “What Does ‘2’ Mean?” p. 139
Sec 3.1: Beyond Numbers, “Correspondence” p. 140.
Sec 3.1: Beyond numbers, “Mindscapes” (activities) pp. 141-144.
Sec. 3.3: The Missing Number, "Unequal Decimals," p.164.
(B) use addition, subtraction, multiplication, and division to solve
problems involving fractions and decimals;
Sec. 3.3: The Missing Number, "Unequal Decimals," p.164.
(C) use models to add, subtract, multiply, and divide integers and connect the actions to algorithms;
Sec 3.2: Comparing the Infinite, "Integers Equal Naturals," pp. 151-152.

§111.23. Mathematics, Grade 7. September 1997 Original Page B-7
(7.6) **Geometry and spatial reasoning.** The student compares and classifies shapes and solids using geometric vocabulary and properties. The student is

expected to:
(B) use properties to classify shapes including triangles, quadrilaterals, pentagons, and circles;
Sec. 3.5: Straightening Up the Circle, pp. 190-198.

(7.7) **Geometry and spatial reasoning.** The student uses coordinate geometry to describe location on a plane. The student is expected to:
(A) locate and name points on a coordinate plane using ordered pairs of integers;
Sec 3.2: Comparing the Infinite, "Integers Equal Naturals," pp. 151-152.

Ch. 111, TEKS for Mathematics. Page B-8 Texas Education Agency
(7.11) **Probability and statistics.** The student understands that the way a set of data is displayed influences its interpretation.
The student is expected to:
(A) select and use an appropriate representation for presenting collected data and justify the selection; and
(B) make inferences and convincing arguments based on an analysis of given or collected data.
(A) – (B):
Sec. 3.4: Travels Toward the Stratosphere of Infinities, "Sets Within Sets," pp.175-179.

(7.12) **Probability and statistics.** The student uses measures of central tendency and range to describe a set of data.
The student is expected to:
(A) describe a set of data using mean, median, mode, and range; and
(B) choose among mean, median, mode, or range to describe a set of data and justify the choice for a particular situation.
(A) – (B):
Sec. 3.4: Travels Toward the Stratosphere of Infinities, "Sets Within Sets," pp.175-179.

(7.13) **Underlying processes and mathematical tools.** The student applies Grade 7 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school.
The student is expected to:
(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
(D) select tools such as real objects, manipulatives, paper/pencil, and

technology or techniques such as mental math, estimation, and number sense to solve problems.
(A) – (D):
Sec 3.1: Beyond Numbers, "Mindscapes" (activities) p. 141-144.
Sec 3.2: Comparing the Infinite, "Mindscapes" (activities) pp. 156-161.
Sec. 3.3: The Missing Number, "Mindscapes" (activities) pp. 168-172.
Sec 3.4: Travels Toward the Stratosphere of Infinities, "Mindscapes" (activities), pp. 185-189.
Sec. 3.5: Straightening Up the Circle, "Mindscapes" (activities) pp. 201-205.

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(b) **Knowledge and skills.**

(7.14) **Underlying processes and mathematical tools.** The student communicates about Grade 7 mathematics through informal and mathematical language, representations, and models.
The student is expected to:
(A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and
(B) evaluate the effectiveness of different representations to communicate ideas.
(A) through (B):
Sec 3.1: Beyond Numbers, "Mindscapes" (activities) p. 141-144.
Sec 3.2: Comparing the Infinite, "Mindscapes" (activities) pp. 156-161.
Sec. 3.3: The Missing Number, "Mindscapes" (activities) pp. 168-172
Sec 3.4: Travels Toward the Stratosphere of Infinities, "Mindscapes" (activities), pp. 185-189.
Sec. 3.5: Straightening Up the Circle, "Mindscapes" (activities) pp. 201-205

(7.15) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions.
The student is expected to:
(A) make conjectures from patterns or sets of examples and nonexamples;
Sec. 3.4: Travels Toward the Stratosphere of Infinities, "Sets Within Sets," pp.175-179.

§111.24. Mathematics, Grade 8.

(b) **Knowledge and skills.**

(8.1) **Number, operation, and quantitative reasoning.** The student understands that different forms of numbers are appropriate for different situations. The student is expected to:
(A) compare and order rational numbers in various forms including integers, percents, and positive and negative fractions and decimals;
Sec 3.2: Comparing the Infinite, "Rational Numbers," pp.152-155.
Sec 3.2: Comparing the Infinite, "Integers Equal Naturals," pp. 151-152.

(B) select and use appropriate forms of rational numbers to solve real-life problems including those involving proportional relationships;
Sec 3.2: Comparing the Infinite, "Rational Numbers," pp.152-155.

(8.2) **Number, operation, and quantitative reasoning.** The student selects and uses appropriate operations to solve problems and justify solutions. The student is expected to:
(A) select and use appropriate operations to solve problems and justify the selections;
Sec 3.1: Beyond Numbers, “What Does ‘2’ Mean?” p. 139.
Sec 3.1: Beyond Numbers, “Correspondence” p. 140.
Sec 3.1: Beyond numbers, “Mindscapes” (activities) pp. 141-144.
(B) add, subtract, multiply, and divide rational numbers in problem situations;
Sec 3.2: Comparing the Infinite, "Rational Numbers," pp.152-155.

(8.3) **Patterns, relationships, and algebraic thinking.** The student identifies proportional relationships in problem situations and solves problems. The student is expected to:
(A) compare and contrast proportional and non-proportional relationships; and
Ch. 3.2: Comparing the Infinite, pp. 145-161.

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(8.5) **Patterns, relationships, and algebraic thinking.** The student uses graphs, tables, and algebraic representations to make predictions and solve problems.
The student is expected to:
(D) locate and name points on a coordinate plane using ordered pairs of rational numbers.
Sec 3.2: Comparing the Infinite, "Rational Numbers," pp.152-155.

(8.10) **Measurement.** The student describes how changes in dimensions affect linear, area, and volume measures.
Sec. 3.5: Straightening Up the Circle, pp. 190-198.

(8.12) **Probability and statistics.** The student uses statistical procedures to describe data. The student is expected to:
(A) select the appropriate measure of central tendency to describe a set of data for a particular purpose;
Sec. 3.4: Travels Toward the Stratosphere of Infinities, "Sets Within Sets," pp.175-179.

(8.14) **Underlying processes and mathematical tools.** The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to:
(A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;

(B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
(C) select or develop an appropriate problem-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
(D) select tools such as real objects, manipulatives, paper/pencil, and technology or techniques such as mental math, estimation, and number sense to solve problems.
(A) – (D):
Sec 3.1: Beyond Numbers, "Mindscapes" (activities) p. 141-144.
Sec 3.2: Comparing the Infinite, "Mindscapes" (activities) pp. 156-161.
Sec. 3.3: The Missing Number, "Mindscapes" (activities) pp. 168-172.
Sec 3.4: Travels Toward the Stratosphere of Infinities, "Mindscapes" (activities), pp. 185-189.
Sec. 3.5: Straightening Up the Circle, "Mindscapes" (activities) pp. 201-205.

(8.16) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusions. The student is expected to:
(A) make conjectures from patterns or sets of examples and nonexamples;
Sec. 3.4: Travels Toward the Stratosphere of Infinities, "Sets Within Sets," pp.175-179.

High School Grades 9-12

§111.31. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades 9-12.

(b) **Foundations for functions**: knowledge and skills and performance descriptions.

(1) The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.
Following are performance descriptions.
(B) The student gathers and records data, or uses data sets, to determine functional(systematic) relationships between quantities.
Sec. 3.4: Travels Toward the Stratosphere of Infinities, "Sets Within Sets," pp.175-179.

(c) **Linear functions**: knowledge and skills and performance descriptions.
Sec. 3.5: Straightening Up the Circle, pp. 190-198.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use

problem-solving, computation in problem-solving contexts, language and communication, connections within and outside mathematics, and reasoning, as well as multiple representations, applications and modeling, and justification and proof.
Sec 3.1: Beyond Numbers, "Mindscapes" (activities) p. 141-144.
Sec 3.2: Comparing the Infinite, "Mindscapes" (activities) pp. 156-161.
Sec. 3.3: The Missing Number, "Mindscapes" (activities) pp. 168-172
Sec 3.4: Travels Toward the Stratosphere of Infinities, "Mindscapes" (activities), pp. 185-189.
Sec. 3.5: Straightening Up the Circle, "Mindscapes" (activities) pp. 201-205

Source: The provisions of this §111.33 adopted to be effective September 1, 1996, 21 TexReg 7371.

§111.34. Geometry (One Credit).

(b) **Geometric structure**: knowledge and skills and performance descriptions.

(B) The student makes and verifies conjectures about angles, lines, polygons, circles, and three- dimensional figures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.
Sec. 3.5: Straightening Up the Circle, pp. 190-205
(4) The student uses a variety of representations to describe geometric relationships and solve problems. Following is a performance description. The student selects an appropriate representation (concrete, pictorial, graphical, verbal, or symbolic) in order to solve problems.

(c) **Geometric patterns:** knowledge and skills and performance descriptions. The student identifies, analyzes, and describes patterns that emerge from two- and three-dimensional geometric figures.

(1) The student uses numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures and solids, and angle relationships in polygons and circles.
Sec. 3.5: Straightening Up the Circle, pp. 190-198

(d) **Dimensionality and the geometry of location**: knowledge and skills and performance descriptions.

(1) The student analyzes the relationship between three- dimensional objects and related two-dimensional representations and uses these representations to solve problems.
Following are performance descriptions.
(A) The student describes, and draws cross sections and other slices of three-dimensional objects.
(B) The student uses nets to represent and construct three-dimensional objects.
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(C) The student uses top, front, side, and corner views of three-dimensional objects to create accurate and complete representations and solve problems.
(A) through (C): Sec. 3.5: Straightening Up the Circle, pp. 190-205

(2) The student understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly.
Following are performance descriptions.
(A) The student uses one- and two- dimensional coordinate systems to represent points, lines, line segments, and figures.
Sec. 3.5: Straightening Up the Circle, pp. 201-205.
(B) The student uses slopes and equations of lines to investigate geometric relationships, including parallel lines, perpendicular lines, and special segments of triangles and other polygons.
Sec. 3.5: Straightening Up the Circle, p. 202

(1) The student extends measurement concepts to find area, perimeter, and volume in problem situations.
Following are performance descriptions.
(B) The student finds areas of sectors and arc lengths of circles using proportional reasoning.
Sec. 3.5: Straightening Up the Circle, pp. 190-198

(2) The student analyzes properties and describes relationships in geometric figures.
Following are performance descriptions.
Sec. 3.5: Straightening Up the Circle, pp. 201-205.
(C) Based on explorations and using concrete models, the student formulates and tests conjectures about the properties and attributes of circles and the lines that intersect them.
Sec. 3.5: Straightening Up the Circle, pp. 190-198
(D) The student analyzes the characteristics of three-dimensional figures and their component parts.
Sec. 3.5: Straightening Up the Circle, pp. 190-205

(f) **Similarity and the geometry of shape**: knowledge and skills and performance descriptions. The student applies the concepts of similarity to justify properties of figures and solve problems.
Sec. 3.5: Straightening Up the Circle, p. 202

§111.36. Mathematical Models with Applications (One-Half to One Credit).

(a) **General requirements**. The provisions of this section shall be implemented beginning September 1, 1998. Students can be awarded one-half to one credit for successful completion of this course. Recommended prerequisite: Algebra I.

(3) The student develops and implements a plan for collecting and analyzing data in order to make decisions. The student is expected to:

(C) determine the appropriateness of a model for making predictions from a given set of data.
Sec. 3.4: Travels Toward the Stratosphere of Infinities, "Sets Within Sets," pp.175-179.

(4) The student uses probability models to describe everyday situations involving chance.
Sec. 3.4: Travels Toward the Stratosphere of Infinities, "Endless Infinities," pp.174-175.